

Ergodic Theory - Week 13

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1 Entropy

- P1.** Calculate the entropy of any periodic system.
- P2.** (a) Calculate the entropy of a rotation by any $\alpha \in \mathbb{T}$ on the torus $\mathbb{T}$.
(b) Calculate the entropy of a rotation by any $\alpha \in \mathbb{T}^d$ on the d -dimensional torus $\mathbb{T}^d$, for any $d \in \mathbb{N}$.
- P3.** Calculate the entropy of the map $T_d : x \rightarrow dx \bmod 1$ on the torus, for any $d \geq 2$. Conclude that any two distinct systems of the form $(\mathbb{T}, \mathcal{B}(T), \lambda, T_d)$, $d \in \mathbb{N}$ are not isomorphic.
- P4.** Calculate the entropy of the Bernoulli shift on $X = \{0, 1, \dots, k\}^{\mathbb{N}}$, where the probability measure in $\{0, \dots, k\}$ is defined by the probability vector $(p_0, p_1, \dots, p_k)$.